Six-Port and Four-Port Reflectometers for Complex Permittivity Measurements at Submillimeter Wavelengths

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Abstract—**The frequency range of six-port reflectometry has been extended into the submillimeter wavelength range. The complex permittivity of low-loss microwave materials has been determined using new six-port and four-port reflectometers developed in the Physikalisch-Technische Bundesanstalt (PTB). These either consist of quasi-optical components or utilize oversized waveguide techniques. Permittivity measurements of materials possessing a wide range of values of ϵ' (2 to 7) and of the loss tangent (0.0003 to 0.03) were carried out at frequencies of about 380 GHz to 390 GHz. Good agreement with published permittivity data is shown.**

I. Introduction

THE NEED FOR reliable dielectric data in the millimeter and submillimeter wavelength range has given rise to increasing measurement activity. At frequencies of 90 GHz and above, homogeneous and isotropic solids of low dielectric loss have been investigated in order to determine their suitability for use in microwave components and quasi-optical elements for new microwave systems that operate in this frequency range.

For precision measurements of the complex permittivity, there are already several techniques in which the dielectric specimen is illuminated by an electromagnetic wave beam several wavelengths in diameter, preferably in free space [1]. As a broad-band method, the Fourier transform spectrometer operated in the dispersive (DFTS) mode, initially developed for the far infrared region, permits measurement of the complex permittivity (or refractive index). Recently, permittivity measurements have been carried out with this method at frequencies down to 100 GHz or less [2]. However, for these frequencies, very sensitive liquid-helium-cooled detectors are required because of the very low power levels of the broad-band noise sources available.

As a precision fixed-frequency method, the resonator technique has been extended to frequencies of up to 140 GHz either using a closed, oversized waveguide cavity [3] or an open quasi-optical bi- or plano-concave mirror system [1]. At submillimeter wavelengths, the design of the tiny coupling structure of the cavity and the manufacture of well-shaped spherical mirrors for the open resonator become difficult. As an alternative, laser-driven quasi-optical two-arm (Mach–Zehnder) interferometers have therefore been designed for transmission measurements at 245 GHz [4]; these are balanced by optical attenuators which, however, cannot be easily traced to attenuation standards.

In this paper, multiport reflectometers developed in the Physikalisch-Technische Bundesanstalt (PTB) and consisting of simple linear microwave networks are described by which the complex permittivity is determined by complex reflectometry. The reflectometers operate at the low end of the submillimeter waveband, i.e., at fixed frequencies of about 380 to 390 GHz. A simple calibration method, which is also explained in detail, covers all the effects of multiple reflections within the reflectometers or between components of the reflectometers and the load. At the low end of the submillimeter waveband, it is valuable to have available a method of complex permittivity measurement whose principle is completely different from the broad-band methods. It has been observed for these frequencies that DFTS measurements of the complex permittivity on the same dielectric specimens carried out in different laboratories showed different results; hence a certain degree of systematic error cannot be excluded here.

One of the reflectometers is designed according to the six-port principle but using a quasi-optical technique; furthermore, two four-port reflectometers of even simpler construction, one using quasi-optical and one using oversized waveguide techniques, have been successfully developed. With these instruments, the relative complex permittivity

$$\epsilon = \epsilon'(1 - j\tan\delta) \qquad (1)$$

of homogeneous isotropic dielectric solid materials of low loss ($\tan\delta = 0.0003$ to 0.03, $j = \sqrt{-1}$) can be measured in terms of the complex input reflection coefficient Γ_l of the dielectric specimen. The specimen is a cylinder of length d with plane parallel flat surfaces which is short-circuited at its back using a highly conductive plane reflector plate.

With six-port reflectometers, the complex reflection coefficient is determined by simple power measurements using four power meters at four ports of a linear passive microwave network of suitable design; expensive superheterodyning of the type used with customary network analyzers is therefore avoided. The six-port method as developed by Hoer and Engen [5] has been realized in both coaxial and standard waveguide techniques, the latter for

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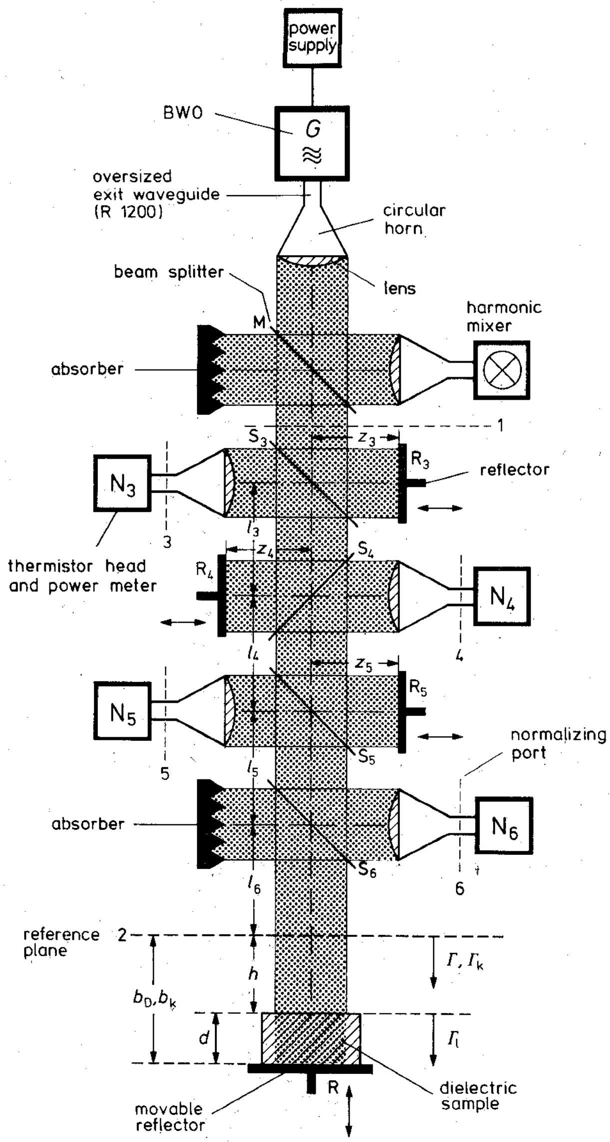

Fig. 1. Block diagram of the quasi-optical submillimeter six-port reflectometer.

frequencies up to 110 GHz. With the new device developed in the PTB and described here, six-port reflectometry has been successfully carried out at submillimeter wavelengths [6].

In this paper, it will also be shown that with phasable reflection loads, a much simpler four-port reflectometer can be used such that only two power meters are required instead of four. For both reflectometer types, the same calibration method can be applied.

II. Experimental Setup

A. Quasi-Optical Six-Port Reflectometer

A block diagram of the device is shown in Fig. 1. The experimental apparatus is shown in Fig. 2. The reflectometer is mounted on a sturdy upright aluminum frame about 500 mm in length. At the top, a microwave source (backward wave oscillator (BWO), G in Fig. 1) generates a coherent microwave signal of about 100 mW power at fixed frequencies of about 380 to 390 GHz. The phase

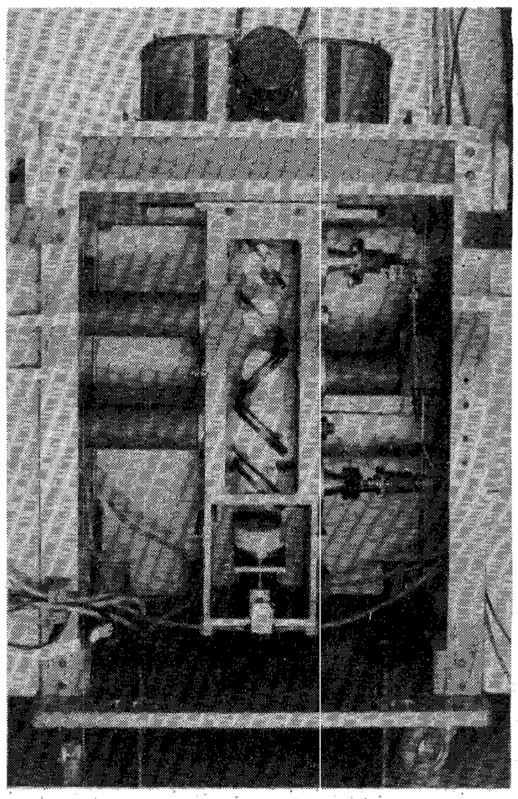

Fig. 2. Front view of the quasi-optical submillimeter six-port reflectometer with polyethylene specimen 80 mm in diameter and about 30 mm in length.

surfaces of the electromagnetic wave are made plane parallel by a horn antenna of circular aperture 50 mm in diameter which is connected to the IEC R1200 standard output waveguide of the source and terminated by a polyethylene lens 50 mm in diameter. Part of the vertical beam is reflected back from the dielectric specimen which is short-circuited by an axially movable plane copper reflector plate R located at the bottom of the frame.

The pattern of the resulting field of standing waves in front of the specimen is sampled by three fixed nondirective probes: 3, 4, and 5. The field probes are verified by three Michelson interferometers, each consisting of the common reflector R, a 45° dielectric beam splitter made from 0.15 mm to 0.2 mm thick Astralan foil (S_3 to S_5), and a plane reflector (R_3 to R_5). About -15 dB of the signals incident on and reflected from the specimen are coupled out into receiving horn antennas located at the sides of the frame. The coupled energy is fed through circular brass waveguides 1 mm in diameter onto the thermistor elements (about 0.3 mm in diameter) of commercial IEC R100 (X-band) standard waveguide bolometer mounts N_3 to N_5 at the "measuring" ports 3, 4, and 5, by which RF power values P_3' to P_5' having typical values of 0.1 mW to 1 mW are measured.

The positions of the side arm reflectors R_3 to R_5 are fixed during calibration and measurement, but can be axially shifted after a change of the measurement frequency in order to maintain a 120° azimuthal distribution of the positions of the q points that characterize the six-port device. This ensures the lowest uncertainty of measurement for reflection coefficients within the unit circle in the complex reflection plane. In order to eliminate errors due to power fluctuations of the source G, it is necessary to measure the generator RF power P_6' at the "normalizing" port 6 using an additional thermistor head N_6 and to divide P_3' to P_5' by P_6'.

The thermistors are connected to commercial power meters that employ dc substitution bridges. A plane absorber foil mounted at 45° to the beam axis, which can be introduced into the beam at port 1, serves as a microwave switch. The bridge voltages as measured with RF power switched on and off are digitized and fed to a desktop calculator which further processes the data.

The beam splitter M serves as a coupler to a commercial IEC R1200 waveguide harmonic mixer which is part of the frequency locking and measuring circuit (not shown in Fig. 1). The locked frequency of the microwave source can be determined with an uncertainty of about ±100 kHz.

The reflector R is mounted on a motor-driven vertical translator fitted with a commercial digital translation measuring system, the digital output of which is also connected to the desktop calculator. The distance b of the reflector to the reference plane at port 2 can be adjusted and measured with a resolution of 0.0001 mm, corresponding to a phase angle of about 0.1° at the frequencies stated.

B. Four-Port Reflectometers

With the four-port reflectometers, the dielectric specimen and the reflector R are axially displaced by known increments and the resulting standing wave pattern in front of the specimen is monitored by only one fixed field probe. Three independent quasi-optical four-port reflectometers are obtained by considering each measuring port of the six-port reflectometer, port 3, 4, or 5, together with the normalizing port 6, as a separate configuration. Alternatively, measurements were carried out using a configuration that included only one Michelson interferometer field probe. The beam splitters of the two other Michelson interferometers which were not in use were removed. The aperture of the antenna was 30 mm in this case. A block diagram of the latter configuration is shown in Fig. 3, where the normalizing port is now numbered 4.

It has been observed with the quasi-optical reflectometers that the calibration parameters show small time-dependent variations due to changes in the distances between the single components which are caused by temperature fluctuations within the large systems. Consequently, a frequent recalibration of these reflectometers is necessary. A small, rigid reflectometer with improved stability against temperature fluctuations which operates in an oversized waveguide has therefore been developed. A block diagram is shown in Fig. 4 and a photograph of the apparatus appears in Fig. 5. The reflectometer consists of two rectangular tellurium copper blocks, both 85 mm in length and 40 mm in width, their plane, lapped surfaces screwed together. The system of oversized crossed waveguides is obtained by milling grooves of rectangular cross section, 2

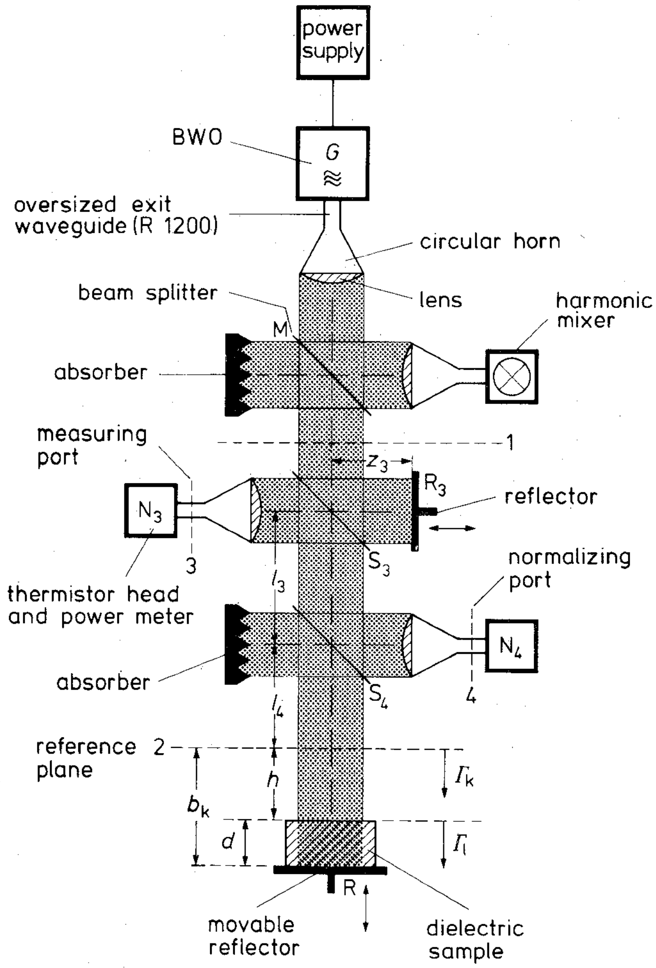

Fig. 3. Block diagram of the quasi-optical submillimeter four-port reflectometer.

mm wide and 1 mm deep (corresponding to IEC R1200 standard waveguide), into the surface of one of the blocks. All copper surfaces are gold-finished. On the upper side, the oversized waveguide is connected via a length of standard R1200 waveguide to the BWO. Beam splitters (M, S_3, and S_4) made from the same foil used with the six-port reflectometer are fixed in the waveguide in narrow 45° grooves cut into the copper blocks. A horn antenna and a lens with a 30 mm aperture (as used with the quasi-optical four-port reflectometer) are connected to the lower end of the waveguide.

The microwave signal of the BWO is partially coupled out by the beam splitter M to the mixer of the frequency-locking circuit. Beam splitters S_3 and S_4 reflect some RF power into the measuring and normalizing ports 3 and 4, respectively, where the power is guided via circular waveguides to the commercial X-band thermistor mounts. The movable short circuit R_3 in the side arm was later replaced by a fixed reflector plate located at the end of the side arm waveguide.

To achieve a certain degree of source–reflectometer decoupling, attenuation foils can be introduced into the waveguide through attenuator slits. RF power is switched on and off with a microwave switch consisting simply of

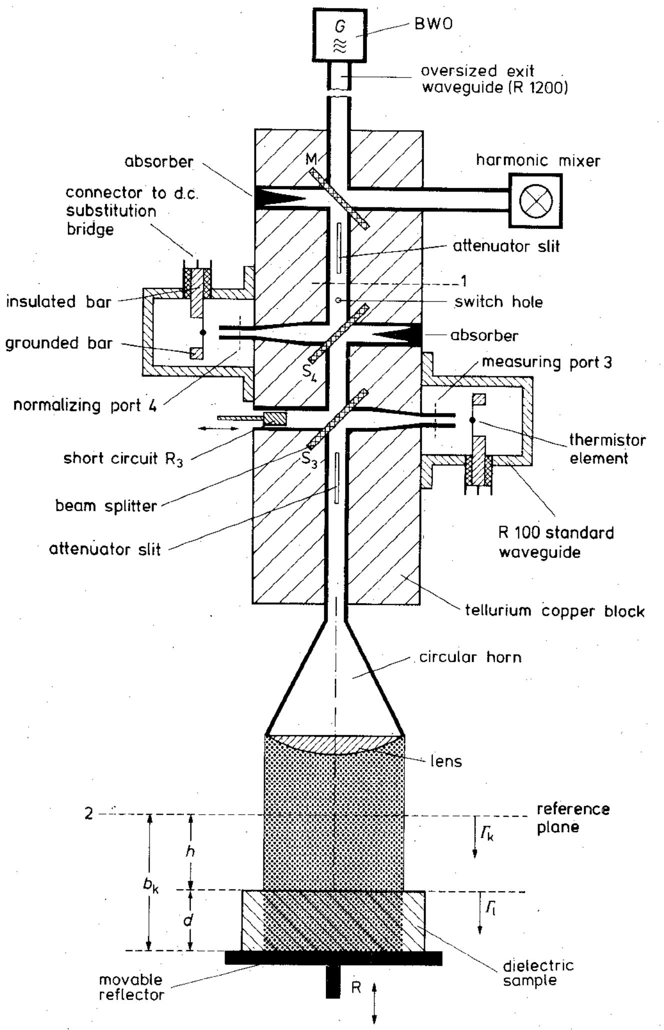

Fig. 4. Block diagram of the submillimeter four-port reflectometer using an oversized waveguide technique.

an absorbing rod about 1.8 mm in diameter. The switch rod is introduced into or withdrawn from the waveguide through a switch hole using a small electromagnetic relay that is controlled by the desktop calculator.

III. Theory

A. General Considerations

The application of multiport reflectometers is not limited to coaxial or waveguide techniques; six-port [7] and four-port reflectometry should also be possible for plane TEM waves in free space. In the case of the six-port reflectometer, it is assumed here that only one wave mode at each of the six ports is propagating and that the reflectometer is completely described by three scalar system equations:

$$P_i = K_i \cdot |\Gamma - q_i|^2 / |1 - \Gamma \cdot \Gamma_g|^2 \tag{2}$$

which relate the measured normalized power values $P_i = P_i'/P_6'$ at each measuring port ($i = 3,4,5$) to the complex reflection coefficient,

$$\Gamma = x + jy \tag{3}$$

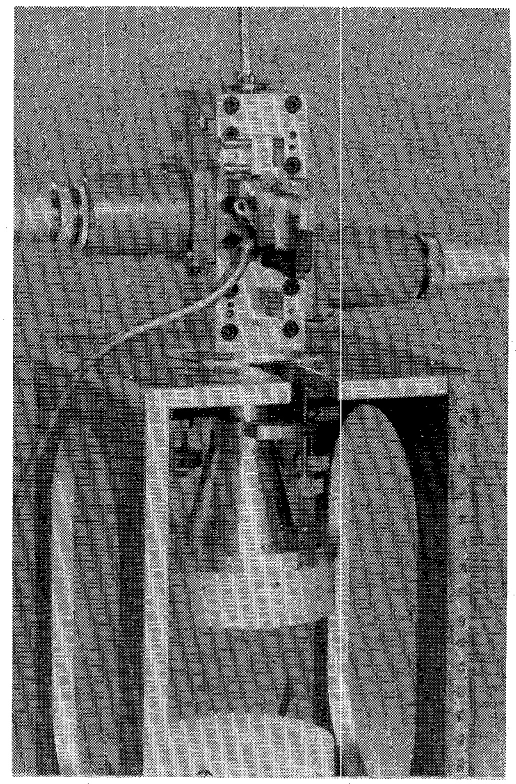

Fig. 5. Front view of the submillimeter four-port reflectometer using an oversized waveguide technique.

at the output reference plane 2 [5]. K_i, Γ_g, and q_i represent the system parameters. K_i are scalar instrument parameters, while

$$\Gamma_g = u + jw \tag{4}$$

is the complex reflection coefficient of the reflectometer observed when looking into output port 2. The parameters

$$q_i = s_i + jt_i \tag{5}$$

determine the so-called complex q points in the reflection plane. Substituting into (2) gives

$$P_i = K_i \cdot \left(x^2 + y^2 + s_i^2 + t_i^2 - 2s_i \cdot x - 2t_i \cdot y\right) / N$$

where

$$N = 1 + \left(x^2 + y^2\right)\left(u^2 + w^2\right) - 2ux + 2wy \qquad \text{for } i = 3,4,5. \tag{6}$$

With K_i, Γ_g, and q_i known from calibration, Γ is determined as the intersection point of three circles [8] with the radii

$$R_i = \sqrt{P_i \cdot K_i} \cdot |1 - q_i \cdot \Gamma_g| / |P_i \cdot \Gamma_g \Gamma_g^* - K_i| \tag{7}$$

and centers (the C points)

$$C_i = \left(P_i \cdot \Gamma_g^* - q_i \cdot K_i\right) / \left(P_i \cdot \Gamma_g \Gamma_g^* - K_i\right) \tag{8}$$

for $i = 3,4,5$, where the asterisk stands for complex conjugate. In the case of the four-port reflectometers only one scalar equation, (2) or (6) (e.g. for $i = 3$), is required for describing the system.

B. Calibration

The system parameters are frequency dependent and have to be determined anew by calibration for each fixed measuring frequency. The calibration method selected allows a separate determination of each set of system parameters pertinent to a certain measuring port ($i=3,4,5$) by a simple regression analysis where a set of $M>=5$ (usually 6) known reflection coefficient values Γ_k (where $k=1,\cdots,M$) is used as a reflection standard. The reflection standard is easily realized by means of the highly conductive reflector R (with the dielectric specimen removed), which acts as a phasable short circuit for the plane TEM waves. $|\Gamma_k|$ is assumed to be 1; therefore

$$\Gamma_k=-\exp(-2\beta b_k) \tag{9}$$

is valid where the phase angles are calculated from the known distances b_k of the reflector R to the reference plane 2 and the phase coefficient of the plane TEM waves in free space as given by

$$\beta=2\pi f/c. \tag{10}$$

Here c is the speed of light in air and f is the known measuring frequency. For each value of b_k, a normalized power value P_k is measured and stored in the desktop calculator.

The system parameters pertinent to the ports $i=3,4,5$ are determined as follows. For simplification, the port index i is omitted here. Index k indicates the position of the reflector R. For each $\Gamma_k=x_k+jy_k$, the system equation (6) becomes

$$P_k=K\left(-2sx_k-2ty_k+1+s^2+t^2\right)\Big/\left(-2ux_k+2wy_k+1+u^2+w^2\right). \tag{11}$$

It is convenient to rewrite (11) as

$$P_k=(a'x_k+b'y_k+c')/(d'x_k+e'y_k+1) \tag{12}$$

for $k=1,\cdots,M$, where the five new parameters are defined by

$$\begin{aligned}a'&=-2sK/Q\\ b'&=-2tK/Q\\ c'&=K(1+s^2+t^2)/Q\\ d'&=-2u/Q\\ e'&=2w/Q\\ Q&=1+u^2+w^2.\end{aligned} \tag{13}$$

Due to the limited accuracy of power measurements, a least-squares method is applied to calculate $a',\cdots,e'$. The most satisfactory of all possible sets of $a',\cdots,e'$ is that which renders the sum of the squares

$$S=\sum_{k=1}^{M}\left(P_k+d'P_kx_k+e'P_ky_k-a'x_k-b'y_k-c'\right)^2 \tag{14}$$

a minimum. A necessary condition for this is that all derivatives $\delta S/\delta a',\cdots,\delta S/\delta e'$ vanish. From this condition we obtain a set of five linear equations which can be solved for $a',\cdots,e'$:

$$\begin{aligned}&d'\sum P_k^2x_k^2+e'\sum P_k^2x_ky_k-a'\sum P_kx_k^2\\ &\quad-b'\sum P_kx_ky_k-c'\sum P_kx_k=-\sum P_k^2x_k\\ &d'\sum P_k^2x_ky_k+e'\sum P_k^2y_k^2-a'\sum P_kx_ky_k\\ &\quad-b'\sum P_ky_k^2-c'\sum P_ky_k=-\sum P_k^2y_k\\ &d'\sum P_kx_k^2+e'\sum P_kx_ky_k-a'\sum x_k^2\\ &\quad-b'\sum x_ky_k-c'\sum x_k=-\sum P_kx_k\\ &d'\sum P_kx_ky_k+e'\sum P_ky_k^2-a'\sum x_ky_k\\ &\quad-b'\sum y_k^2-c'\sum y_k=-\sum P_ky_k\\ &d'\sum P_kx_k+e'\sum P_ky_k-a'\sum x_k\\ &\quad-b'\sum y_k-c'M=-\sum P_k\end{aligned} \tag{15}$$

with the sums to be taken for $k=1,\cdots,M$. The set K,s,t,u,w is then calculated using

$$Q=2\left(1-\sqrt{1-(d'^2+e'^2)}\right)\Big/(d'^2+e'^2) \tag{16}$$

$$K=Q\left(c'\pm\sqrt{c'^2-(a'^2+b'^2)}\right)\Big/2 \tag{17}$$

where the positive sign in (17) is valid if $|q|<1$, and the negative sign is valid if $|q|>1$. For the submillimeter reflectometers described here, $|q|>1$ is always valid, because attenuation exists between the (lowest) Michelson interferometer probe and reference plane 2 (created by the beam splitter appertaining to the normalizing port in the case of the quasi-optical configurations or by the attenuator and horn antenna in the case of the waveguide four-port reflectometer). Finally we obtain from (13)

$$\begin{aligned}s&=-a'Q/(2K)\\ t&=-b'Q/(2K)\\ u&=-d'Q/2\\ w&=e'Q/2.\end{aligned} \tag{18}$$

The whole procedure can then be repeated for the next measuring port, using the same set of reflection values x_k, y_k ($k=1,\cdots,M$). The parameters u and w are determined anew each time, so that at the end of the calibration routine, besides the parameters K_i and q_i, three reflection parameters

$$\Gamma_{gi}=u_i+jw_i \tag{19}$$

are available for the three measuring ports $i=3$, 4, and 5.

Typical values of the system parameters for the different reflectometers are given in Table I, showing that with the six-port reflectometer, a 120° distribution of the q points can be approximatively achieved. The values of $|q|$ vary from about 2.7 to 1.7, so that maximum power variations of about 7 dB to 12 dB occur at the different measuring ports. The three values of Γ_{gi} given in Table I, which are, in theory, equal, differ from one another (typically by about ±0.01 for the modulus and by about $\pm15°$ for the phase angle). This effect is attributed to a certain malad-

TABLE I
TYPICAL VALUES OF SYSTEM PARAMETERS AT SELECTED FREQUENCIES FOR THE DIFFERENT MEASURING PORTS OF THE QUASI-OPTICAL SIX-PORT REFLECTOMETER AND FOR THE QUASI-OPTICAL AND WAVEGUIDE FOUR-PORT REFLECTOMETERS

device	frequency GHz	K	|q|	arg(q)°	$\|\Gamma_g\|$	arg(Γ_g)°
six-port						
port 3	392,309	0,342	2,715	164,7	0,048	287,2
port 4	392,309	0,307	2,484	39,6	0,064	266,7
port 5	392,309	0,977	1,703	293,7	0,051	260,9
quasi-opt.	380,251	0,243	2,145	90,3	0,036	92,4
four-port	391,766	0,354	1,715	225,1	0,012	300,9
waveguide four-port	380,086	0,789	1,909	80,9	0,177	263,3

justment of the optical components, mainly of the reflector R. Due to a small misalignment of the axes of the rays incident on and reflected from R, the wave beams coupled out of the reflected beam into the side arm receiving antennas (Fig. 1) become laterally shifted with respect to the antenna apertures, by amounts which depend on the varying distances b of the reflector R to the reference plane 2 and on the distance from R to the three beam splitters. This may cause a change in the RF power coupled out from the reflected beam into the thermistor.

With the waveguide four-port reflectometer, $|\Gamma_g|$ is considerably greater than with the quasi-optical devices.

C. Evaluation of Six-Port Reflectometer Measurements

To perform reflection measurements, a dielectric specimen of known length d is placed on the reflector R, which is positioned at a known fixed distance $b_D = h + d$ from the reference plane 2. Normalized power values P_i are determined for the measuring ports ($i = 3,4,5$) of the six-port reflectometer. A regression method, similar to that used with the calibration, is applied to calculate the complex reflection coefficient (3) with respect to the reference plane 2. The three system equations (6) with known system parameters are rearranged to yield error functions

$$F_i = r^2 \cdot A_i + xB_i + yC_i - D_i \tag{20}$$

($i = 3,4,5$), where

$$\begin{aligned} r^2 &= x^2 + y^2 \\ A_i &= P_i \cdot (u_i^2 + w_i^2) - K_i \\ B_i &= 2(K_i \cdot s_i - P_i \cdot u_i) \\ C_i &= 2(K_i \cdot t_i + P_i \cdot w_i) \\ D_i &= K_i(s_i^2 + t_i^2) - P_i. \end{aligned} \tag{21}$$

The sum of the squares of F_i,

$$S = \sum_i (F_i)^2 \tag{22}$$

should be a minimum. The quantities r^2, x, and y are considered to be independent. From the necessary condition that the derivatives $\delta S/\delta(r^2), \cdots, \delta S/\delta y$ vanish, a set of three linear equations

$$\begin{aligned} r^2\sum A_i^2 + x\sum A_iB_i + y\sum A_iC_i &= \sum A_iD_i \\ r^2\sum A_iB_i + x\sum B_i^2 + y\sum B_iC_i &= \sum B_iD_i \\ r^2\sum A_iC_i + x\sum B_iC_i + y\sum C_i^2 &= \sum C_iD_i \end{aligned} \tag{23}$$

results (with the sums to be taken for $i = 3,4,5$) which can be solved for r^2, x, and y. The input reflection coefficient of the short-circuited dielectric specimen is then calculated as

$$\Gamma_l = \Gamma \cdot \exp(j2\beta h). \tag{24}$$

D. Evaluation of Four-Port Reflectometer Measurements

With the dielectric specimen in place on the reflector R, six normalized power values P_k are determined at six known values of the distance b_k of the reflector R to the reference plane 2 ($k = 1,\cdots,6$). Values of b_k are chosen such that the difference between adjacent values of b_k is

$$|b_{k+1} - b_k| \simeq \lambda/12 \tag{25}$$

where λ is the wavelength of the TEM wave in free space.

The system equation (6) for port $i = 3$ is taken as the basis for the evaluation of the complex reflection coefficient. For the six power measurements ($k = 1,\cdots,6$), we obtain a set of six equations (port index i omitted):

$$\begin{aligned} P_k &= K(x_k^2 + y_k^2 + s^2 + t^2 - 2sx_k - 2ty_k)/N_k \\ N_k &= 1 + (x_k^2 + y_k^2)(u^2 + w^2) - 2ux_k + 2wy_k. \end{aligned} \tag{26}$$

In these equations,

$$\Gamma_k = x_k + jy_k = \Gamma_l \cdot \exp[-j2\beta(b_k - d)] \tag{27}$$

is the complex reflection coefficient in the reference plane 2 and

$$\Gamma_l = x + jy = r \cdot \exp(j \cdot \arg(\Gamma_l)) \tag{28}$$

is the desired input reflection coefficient of the short-circuited dielectric specimen of length d.

We define error functions for $k = 1,\cdots,6$:

$$F_k = r^2 \cdot A_k + x_kB_k + y_kC_k - D_k \tag{29}$$

where

$$\begin{aligned} r^2 &= x_k^2 + y_k^2 \\ A_k &= P_k \cdot (u^2 + w^2) - K \\ B_k &= 2(Ks - P_ku) \\ C_k &= 2(Kt + P_kw) \\ D_k &= K(s^2 + t^2) - P_k. \end{aligned} \tag{30}$$

Using (27) and (28) we obtain

$$F_k = r^2 \cdot A_k + xE_k + yG_k - D_k \tag{31}$$

where

$$\begin{aligned} E_k &= B_k \cdot \cos[2\beta(b_k - d)] - C_k \sin[2\beta(b_k - d)] \\ G_k &= C_k \cos[2\beta(b_k - d)] + B_k \sin[2\beta(b_k - d)]. \end{aligned} \tag{32}$$

As in (20), r^2, x, and y are regarded as independent quantities.

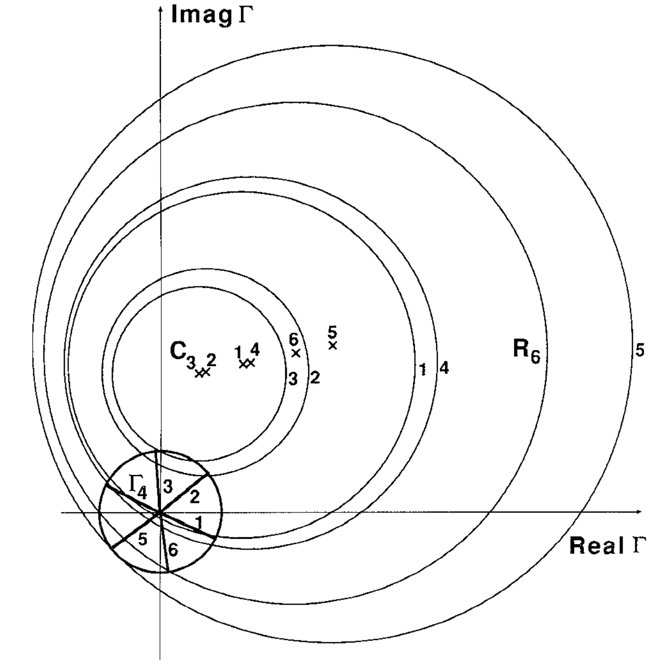

Fig. 6. Complex reflection plane. Values of Γ_k fitting to the circles of radii R_k and centers C_k which represent the power values P_k measured at the reflector positions b_k, for a 5.531 mm thick alumina specimen and with known system parameters as listed in Table I for the waveguide four-port reflectometer ($k=1,\cdots,6$).

The sum of squares for the F_k, taken for $k=1,\cdots,6$, should be a minimum. From the necessary condition that its derivatives with respect to r^2, x, and y vanish, we obtain a set of three linear equations which can be solved for r^2, x, and y:

$$\begin{aligned} r^2\sum A_kE_k + x\sum E_k^2 + y\sum E_kG_k &= \sum E_kD_k \\ r^2\sum A_kG_k + x\sum E_kG_k + y\sum G_k^2 &= \sum G_kD_k \\ r^2\sum A_k^2 + x\sum A_kE_k + y\sum A_kG_k &= \sum A_kD_k \end{aligned} \tag{33}$$

with the sums to be taken for $k=1,\cdots,6$.

The application of the noniterative least-squares method is illustrated in Fig. 6 for a dielectric measurement of an alumina ceramic specimen with $d=5.531$ mm in length. The set of system parameters given in Table I for the waveguide reflectometer was utilized here. Six circles with radii R_k and centers C_k ($k=1,\cdots,6$) are obtained in the complex reflection plane. These are described according to (7) and (8) (with $K_i=K$, $q_i=q$ and the port index i replaced by k here). The circle numbered k represents the geometric locus of the unknown reflection coefficient Γ_k, with known phase angle differences for adjacent values of Γ_k:

$$\arg(\Gamma_{k+1})-\arg(\Gamma_k)=2\beta(b_{k+1}-b_k). \tag{34}$$

By means of the least-squares calculation, the moduli $|\Gamma_k|$ and the phases $\arg(\Gamma_k)$ of the desired reflection coefficients are varied until each Γ_k fits into its locus circle as closely as possible. As can be seen in Fig. 6, this fit is not perfect, due to measurement noise and imperfect calibration or because wave propagation in some additional spurious modes may occur at the reflectometer ports.

E. Calculation of the Complex Permittivity

The complex permittivity is determined by first solving the Roberts–von Hippel equation:

$$(1+\Gamma_l)/(1-\Gamma_l)=j\beta d\cdot\tanh(\gamma_\epsilon\cdot d)/(\gamma_\epsilon\cdot d) \tag{35}$$

with respect to the complex propagation coefficient

$$\gamma_\epsilon=\alpha_\epsilon+j\beta_\epsilon \tag{36}$$

for the TEM wave in the dielectric. The quantities ϵ' and $\tan\delta$ are then determined according to

$$\begin{aligned} \epsilon' &= (\beta_\epsilon^2-\alpha_\epsilon^2)/\beta^2 \\ \tan\delta &= 2\cdot\alpha_\epsilon\beta_\epsilon/(\epsilon'\beta^2). \end{aligned} \tag{37}$$

IV. Measurements

A. Choice of Specimens

At some fixed frequencies between 380 and 390 GHz, measurements of the complex permittivity have been performed on specimens of fused silica, polyethylene, cross-linked polyethylene (Rexolite), glass ceramic (Macor), alumina ceramics, beryllia, and crystal quartz, at a room temperature of 19.7°C±0.2°C and a humidity of about 30 percent. The length d of the specimens was measured using a digital gauge with a resolution of 0.0001 mm. Length values varied from about 1.7 mm to 30 mm (i.e., about 75 half-wavelengths in the latter case). The range of measured reflection coefficient $|\Gamma_l|$ was about 0.2 to 0.95.

B. Uncertainty of Measurement

Uncertainties of measurement are introduced when the reflected TEM wave beam is tilted if the upper surface of the specimen is not perfectly parallel to the reflector R. With the quasi-optical reflectometers, the angle of the rays incident on the receiving horn antennas will be changed when the specimen is placed on the reflector. With the waveguide reflectometer, spurious wave modes may be excited at the discontinuities of the oversized waveguide.

If the lower surface of the specimens is not plane, the remaining air gap between lower surface and reflector causes an additional uncertainty in the permittivity measurement which is approximately of the same magnitude as the uncertainty introduced by that of the measurement of the specimen length d.

Some of the specimens measured were lapped and polished flat such that upper and lower surfaces were parallel to within 0.001 mm. The best reproducibility of measurements with varying azimuthal orientation of the sample was obtained with such specimens. For well-shaped samples, the estimated relative uncertainties were $|\Delta\epsilon'|<0.001$ and about ±12 percent for the loss tangent $\tan\delta$. These correspond essentially to the uncertainty in the reflection coefficient measurement of about ±0.02 for the modulus and ±2° for the phase angle (all uncertainties correspond to a confidence level of 95 percent).

TABLE II
COMPARISON OF REFLECTION COEFFICIENT VALUES OBTAINED FOR FUSED SILICA SPECIMENS AT 392.7 GHz USING THE QUASI-OPTICAL SIX-PORT AND FOUR-PORT REFLECTOMETERS

length d mm	device (No. of port)	$\lvert \Gamma_l \rvert$	arg(Γ_l)°
29,936	six-port	0,717	318
	four-port (3)	0,723	319
	four-port (4)	0,715	318
	four-port (5)	0,716	319
19,9885	six-port	0,738	268
	four-port (3)	0,747	267
	four-port (4)	0,736	268
	four-port (5)	0,723	269
9,981	six-port	0,900	226
	four-port (3)	0,905	226
	four-port (4)	0,900	226
	four-port (5)	0,895	226
30,042	six-port	0,485	160
	four-port (3)	0,496	161
	four-port (4)	0,486	159
	four-port (5)	0,479	159
19,977	six-port	0,702	264
	four-port (3)	0,701	264
	four-port (4)	0,704	264
	four-port (5)	0,699	264

TABLE III
COMPARISON OF PERMITTIVITY VALUES OBTAINED FOR VARIOUS ALUMINA SPECIMENS AND FOR FUSED SILICA AT DIFFERENT FREQUENCIES USING THE SIX-PORT AND FOUR-PORT REFLECTOMETERS OR AN OPEN SEMICONFOCAL RESONATOR

material	frequency GHz	method of measurement	ϵ'	10^3.tan δ
alumina (a)	380,04	quasi-optical four-port	9,699	1,43
	380,08	waveguide four-port	9,696	1,48
	30 - 40	open resonator [9]	9,699	----
alumina (b)	380,04	quasi-optical four-port	9,698	1,52
	380,08	waveguide four-port	9,691	1,42
	30 - 40	open resonator [9]	9,690	----
alumina (c)	380,04	quasi-optical four-port	9,685	1,36
	380,08	waveguide four-port	9,678	1,02
	30 - 40	open resonator [9]	9,670	----
fused silica	392,70	six-port	3,810	1,28
	391,70	quasi-optical four-port	3,811	1,15
	380,04	quasi-optical four-port	3,812	1,28
	380,08	waveguide four-port	3,811	1,18

TABLE IV
PERMITTIVITY MEAN VALUES OBTAINED FOR DIFFERENT MATERIALS WITH THE SIX-PORT AND THE WAVEGUIDE (WG) AND QUASI-OPTICAL (QO) FOUR-PORT REFLECTOMETERS, COMPARED WITH VALUES MEASURED BY ABSORPTIVE (AFTS) AND DISPERSIVE (DFTS) FOURIER TRANSFORM SPECTROMETRY (FTS) AND OTHER METHODS

material	frequency GHz	method of measurement	ϵ'	10^4.tan δ
fused silica	392,7	six-port reflectometer	3,801	12,8
	392,7	AFTS [10]	-----	13,2 1)
polyethylene #1	392,7	six-port reflectometer	2,343	3,72
	392,7	AFTS [10]	-----	3,60 1)
Rexolite	380-390	WG and QO	2,532	27,4
	380	AFTS [11]	-----	28,3 1)
	380	DFTS [11]	2,535	29,1
	380	FTS [11]	2,538	23,7
	140	open resonator [11]	2,535	----
	245	interferometer [11]	2,534	----
Macor	380-390	WG and QO	5,664	269
	380	DFTS [11]	5,650	285
	380	FTS [11]	5,631	270
	30	open resonator [11]	5,664	---
polyethylene #2	380-390	WG and QO	2,319	----
	380	DFTS [11]	2,326	----
	380	FTS [11]	2,314	----
beryllia	380-390	WG and QO	6,693	11,5
	380	DFTS [11]	6,696	13,6
	380	FTS [11]	6,693	6,3
	140	open resonator [11]	6,695	----
quartz crystal	380-390	WG and QO	4,435	----
	380	DFTS [11]	4,436	----
	380	FTS [11]	4,435	----

1) loss tangent values are calculated from power absorption coefficient values and from values of ϵ' as measured by the six- or four-port reflectometers

TABLE V
COMPARISON OF ϵ' DATA OBTAINED USING THE SIX-PORT REFLECTOMETER OR RESONATOR METHODS

material	frequency GHz	method of measurement	ϵ'
alumina	72,0	open resonator [12]	9,743 ± 0,014
	143,9	open resonator [12]	9,740 ± 0,014
	392,7	six-port	9,749 ± 0,003
fused silica	12,4	cavity resonator [13]	3,809 ± 0,003
	35,0	cavity resonator	3,808 ± 0,006
	392,7	six-port	3,810 ± 0,001
polyethylene	10,0	cavity resonator [13]	2,341 ± 0,002
	35,0	cavity resonator	2,341 ± 0,004
	392,7	six-port	2,343 ± 0,001

C. Results

Tables II and III give comparison data obtained with the six-port and four-port reflectometers. In Table II, Γ_l data were obtained for fused silica specimens of different lengths d using either the complete six-port reflectometer or three different four-port configurations; the latter included the common normalizing port and only one of measuring ports 3, 4, or 5 of the original six-port configuration. It is evident that the differences in Γ_l, obtained with six- and four-port reflectometers, were always smaller than the stated uncertainty of measurement for the six-port reflectometer itself. This was also true for polyethylene measurements. Table III gives dielectric data obtained for various specimens of alumina and for fused silica using the waveguide four-port and the quasi-optical six-port and four-port reflectometers (the latter with the beam splitters not in use removed). Good agreement in the data is apparent.

Table IV gives dielectric data on various materials covering a wide range of ϵ' (about 2 to 7) and $\tan\delta$ (about 0.0003 to 0.03) obtained with both six-port and four-port reflectometers. These agree well with values taken from the literature as well as with those extrapolated from results of an international comparison measurement exercise involving 13 laboratories in five countries [11]. Loss tangent data of quartz crystal and polyethylene are not included due to high scatter of the measurement data. This was attributed to poor specimen shape for the polyethylene sample and the very low loss tangent for quartz. For Rexolite, the ϵ' value of 2.532 at 380 GHz agrees well with a value of 2.5322 measured at 245 GHz [4]. Values of ϵ' for alumina, fused silica and polyethylene have also been compared in

Table V with values obtained at lower frequencies using resonator methods. For these low-loss materials the variation of ϵ' with frequency in the spectrum range considered should be smaller than 0.001. The ϵ' data on alumina, compiled in Table III, also agree well with data measured in the 30 to 40 GHz range [9].

V. Conclusion

It has been demonstrated that the principle of six-port reflectometry can also be applied at millimeter-to-submillimeter wavelengths, yielding accurate data on the complex permittivity of low-loss solid materials covering large ϵ' and loss tangent ranges. Good agreement with data published in the literature demonstrates that the six-port and four-port reflectometers are valuable alternatives to other methods, such as broad-band Fourier transform spectrometry. Moreover, the equivalence of the simple waveguide four-port reflectometer (which is preferred because of its easy handling and high stability against temperature fluctuations) to the quasi-optical reflectometers has been shown. The applicability of the waveguide four-port reflectometer for dielectric measurements is not limited to the frequency range in which it was tested. Measurements in the 90 GHz region are planned for the near future.

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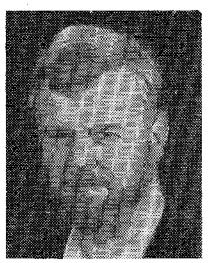

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